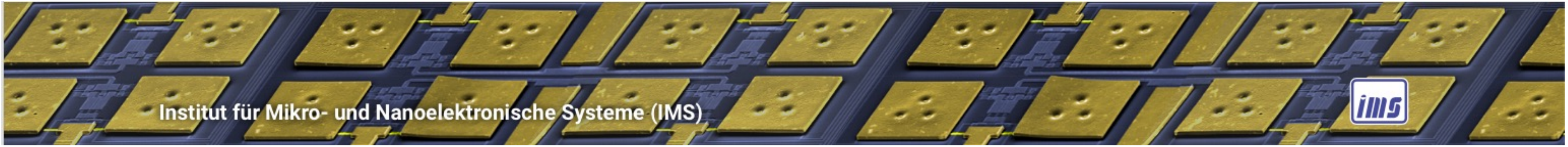
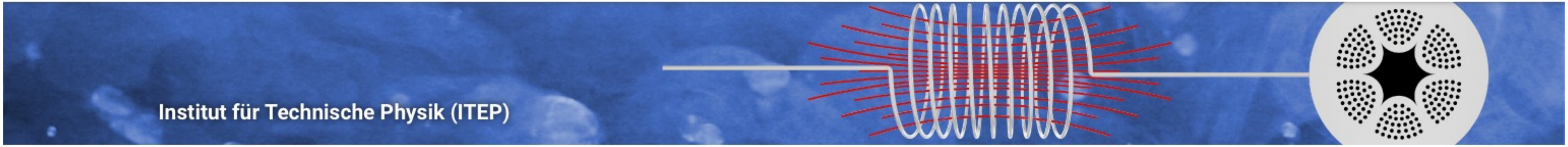
Karlsruher Institut für Technologie

Superconductivity for Engineers

Prof. Dr. Sebastian Kempf, Prof. Dr. Bernhard Holzapfel

Summer term 2021





(Preliminary) Schedule

	Day	Date	Lecture / Tutorial	Day	Date	Lecture / Tutorial
1	Mon	21-04-12	Lecture 1 (SK)	Wed	21-04-14	
2	Mon	21-04-19	Lecture 2 (BH)	Wed	21-04-21	
3	Mon	21-04-26	Lecture 3 (SK)	Wed	21-04-28	Tutorial 1 (IMS)
4	Mon	21-05-03	Lecture 4 (SK)	Wed	21-05-05	
5	Mon	21-05-10	Lecture 5 (SK)	Wed	21-05-12	~~Tutorial 2 (IMS)~~
6	Mon	21-05-17	Lecture 6 (SK)	Wed	21-05-19	Tutorial 2 (IMS)
7	Mon	21-05-24	---	Wed	21-05-26	
8	Mon	21-05-31	Lecture 7 (BH)	Wed	21-06-02	Tutorial 3 (IMS)
9	Mon	21-06-07	Lecture 8 (BH)	Wed	21-06-09	Tutorial 4 (ITEP)
10	Mon	21-06-14	Lecture 9 (BH)	Wed	21-06-16	
11	Mon	21-06-21	Lecture 10 (BH)	Wed	21-06-23	Tutorial 5 (ITEP)
12	Mon	21-06-28	Lecture 11 (BH)	Wed	21-06-30	
13	Mon	21-07-05	Lecture 12 (BH)	Wed	21-07-07	Tutorial 6 (ITEP)
14	Mon	21-07-12	Lecture 13 (SK)	Wed	21-07-14	
15	Mon	21-07-19	Lecture 14 (SK)	Wed	21-07-21	Tutorial 7 (IMS, ITEP)

(Preliminary) Lecture content

Lecture 1:	(SK)	Introduction and overview
Lecture 2:	(BH)	Superconductor applications
Lecture 3:	(SK)	Normal metals and properties of the normal conducting state
Lecture 4:	(SK)	Perfect conductor, ideal diamagnetism, Two-Fluid-Model, London theory
Lecture 5:	(SK)	Disordered superconductors, Pippard theory, microwave properties
Lecture 6:	(SK)	BCS theory
Lecture 7:	**(BH)**	**Type-II superconductors, Current transport**
Lecture 8:	**(BH)**	**Bean Model, Ginzburg-Landau theory**
Lecture 9:	**(BH)**	**GL theory, intermediate state**
Lecture 10:	**(BH)**	**Critical currents, pinning and superconducting permanent magnets**
Lecture 11:	**(BH)**	**Critical currents, pinning and superconducting permanent magnets**
Lecture 12:	**(BH)**	**ac-losses, electrical stabilization and thermal aspects**
Lecture 13:	(SK)	Josephson junctions and SQUIDs
Lecture 14:	(SK)	Josephson junctions and SQUIDs

Flux distribution in real type II superconductors

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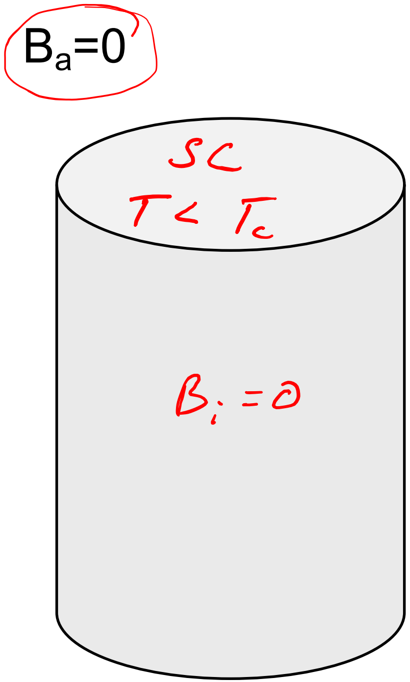
The Bean Model



$j_c(B, T)$ → depend on defect structure of the material

Assumption:

1.) j_c = const ≠ f(B)

2.) $j = j_c$

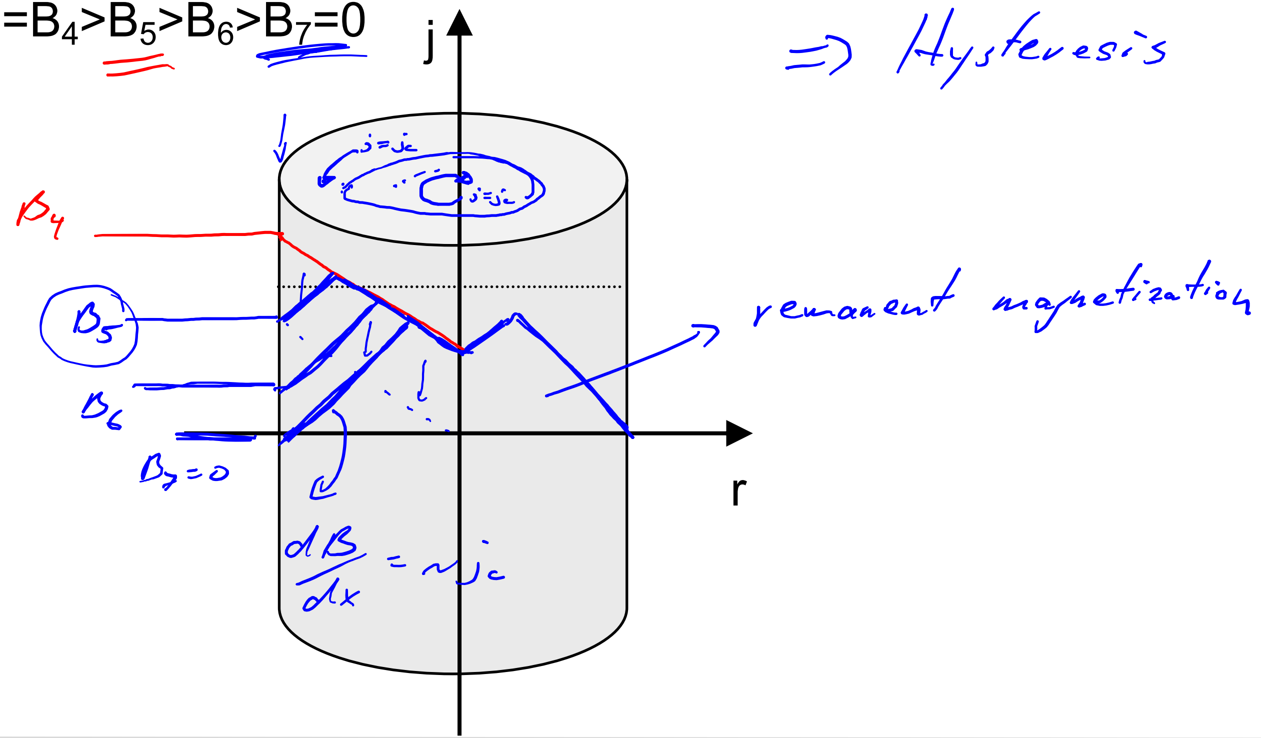
or

$j = 0$

Flux distribution in real type II superconductors

The Bean Model

$B=B_4>B_5>B_6>B_7=0$

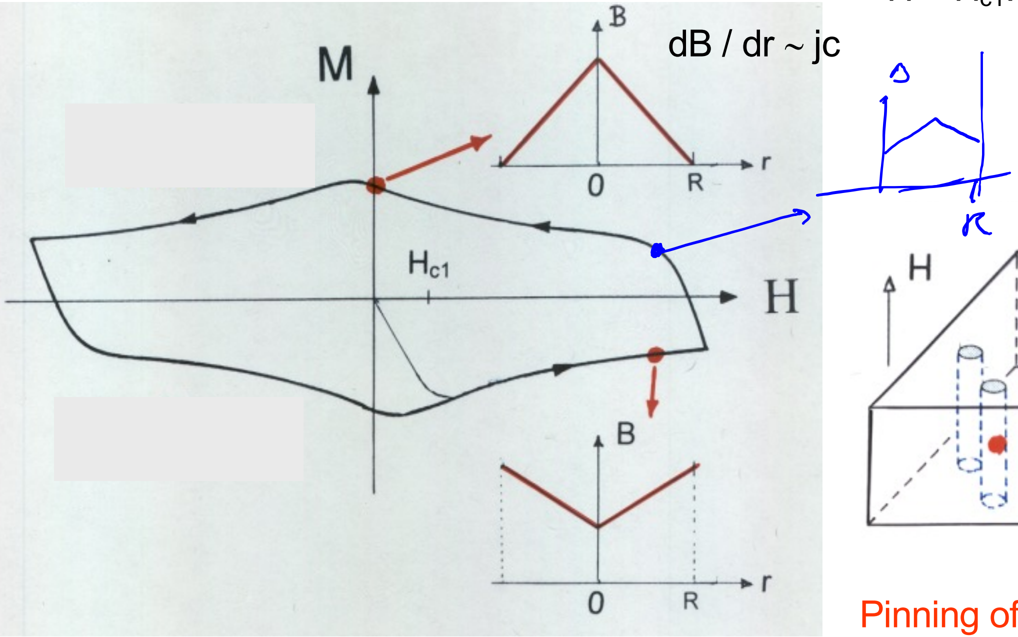


Hysteresis of type II superconductors

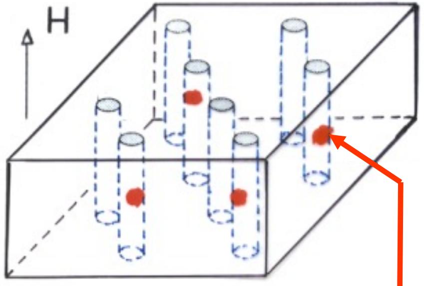
<u>*Magnetization and flux profile*</u>

$H > H_{c1}$: ⇨ flux penetration

dB / dr ~ jc



$\Phi_0 = 2*10^{-15}$ Tm^2
ξ = 1,8 nm



Pinning of flux lines by defects (pinningcenter)

Ginzburg-Landau-Theory: reminder

→ **Phenomenological theory, no need for microscopic understanding; but might be deduced from microscopic theories** (form BCS theory by Gorkov in 59)

→ **Can deal with non-uniform superconducting states** (Shubnikov phase - flux distribution; proximity effect; fluctuations)

→ **Based on an earlier theory of Landau on phase transitions in general**

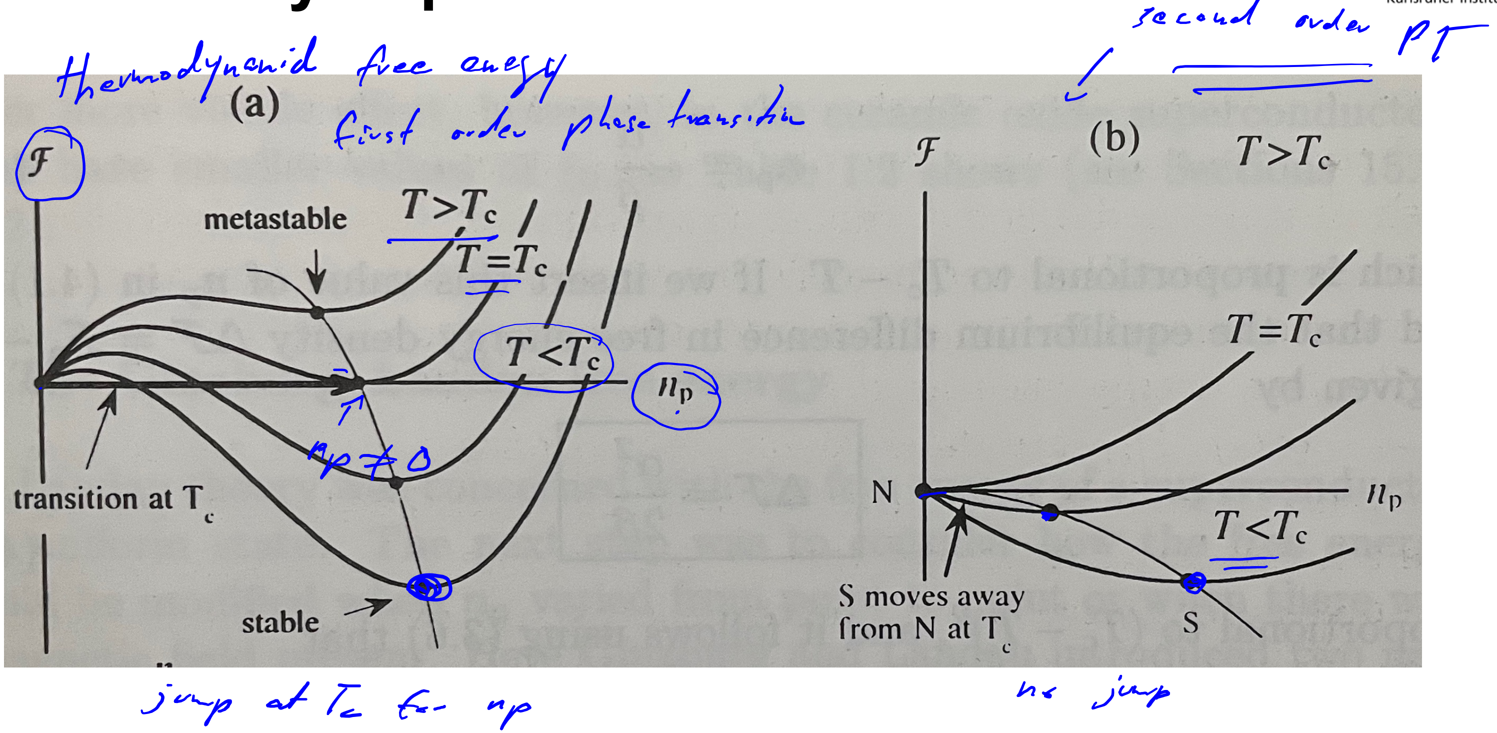
→ **The relevant thermodynamic function depend on the order parameter and the form of this function determines the degree of the phase transition**

→ **For superconductivity: macroscopic wave function is complex order parameter**

→ **Expansion of** $\mathcal{F}(n_p, T)$ **around minimum in powers of** n_p **and** $(T\text{-}T_c)$

→ **GLAG is valid around** T_c **!**

Landau-Theory of phase transitions



Landau-Theory of phase transitions

→ For a second order phase transition $\mathcal{F}(n_p, T)$ is analytic, shows a minimum at T_c and n_p=0 and can be expanded around this point in powers of n_p and (T-T_c)

→ GLAG is valid around T_c !

$$\mathcal{F}(n_p, T) = \mathcal{F}_n(T) + \alpha(T) n_p + \tfrac{1}{2}\beta(T) n_p^2 + \ldots.$$

$$\frac{B_c^2}{2\mu_0} = \Delta\mathcal{F} = \mathcal{F}_n - F_s = \frac{\alpha^2}{2\beta}$$

Minimum of $\mathcal{F}(n_p, T)$:

$$d\mathcal{F}/dn_p = 0 \rightarrow \quad \alpha + \beta n_p = 0 \quad d.h. \;\; n_p = -\frac{\alpha}{\beta}$$

$$B_c(T) = -\sqrt{\frac{\mu_0}{\beta}}\,\alpha(T)$$

Landau-Theory of phase transitions

$n_p = -\frac{\alpha}{\beta}$

$B_c(T) = -\sqrt{\frac{\mu_0}{\beta}\alpha(T)}$

both are proportional to $T_c - T$

$\Delta\mathcal{F} = \mathcal{F}_n - F_s = \frac{\alpha^2}{2\beta}$

$\Delta S = \frac{\partial \Delta F}{\partial T}$ and $\Delta C = T\frac{\partial \Delta S}{\partial T} = T_c\left[\frac{d^2}{dT^2}\left(\frac{\alpha^2}{2\beta}\right)\right]_{T_c}$

from BCS theory $\Delta C = 1.43\gamma T_c \Rightarrow$ $\alpha^2/_\beta = 1.43\gamma(T_c - T)^2$

→ This is first relation involving α and β and *microscopic* theory !

The Ginzburg-Landau free energy

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→ Incorporation of spatial inhomogeneities and magnetic fields

→ Introduction of complex order parameter $\psi(r)\ with\ n_p = \psi^*\psi$

→ "Empirical" construction of F in such a way that it is both ***real*** and invariant under a ***gauge transformation***

$$F = F_n + \int_V \left[\alpha\psi^*\psi + \frac{1}{2}\beta(\psi^*\psi)^2 + \frac{\hbar^2}{4m}\left|\boldsymbol{\nabla}\psi - \frac{2e}{i\hbar}\boldsymbol{A}\psi\right|^2\right] \mathrm{dV} + \int \frac{1}{2\mu_0}(B - B_E)^2\, dV$$

The Ginzburg-Landau equations

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→ Next step is to minimize GL free energy with respect to arbitrary small changes $d\psi(r)$
→ Results in **first Ginzburg-Landau equation**

$$\frac{1}{2m}(-i\hbar\boldsymbol{\nabla}+2e\boldsymbol{A})^2\psi+(\alpha+\beta\psi\psi^*)\psi=0$$

→ Minimizing GL free energy with respect to arbitrary small changes $\boldsymbol{A}$
→ Results in **second Ginzburg-Landau equation**

$$J_s=\frac{ie\hbar}{m}(\psi^*\nabla\psi-\psi\nabla\psi^*)-\frac{4e}{m}A\psi^*\psi$$

Penetration depth and coherence length

→ Second GL equation for constant ψ is identical to London theory
→ **Ginzburg-Landau expression for penetration depth**

$$\lambda = \sqrt{m\beta / 4\mu_0 e^2 |\alpha|}$$

→ First GL equation contains second characteristic length scale
→ Ginzburg-Landau expression for **coherence length**, which characterizes the spatial change characteristics of ψ (e.g. exponential decay of ψ from a point of ψ disturbance)

$$\xi_{GL} = \sqrt{\hbar^2 / 2m|\alpha|}$$

rewrite

$$-\xi_{GL}^2 \left(\frac{\boldsymbol{\nabla}}{i} + \frac{2e}{\hbar}\boldsymbol{A}\right)^2 \psi + \psi - |\psi|^2\psi = 0$$

Penetration depth and coherence length

→ Both GL characteristic length scales have the same α dependence
→ **λ and ξ show the same T-dependence, both diverge at T_c**

$$\xi_{GL}(T) = \frac{\xi_{GL}(0)}{\sqrt{1 - T/T_c}} \qquad \lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1 - T/T_c}}$$

→ The ratio $\kappa = \lambda / \xi$ is T-independent and relevant to separate type I and type II SC

$$\kappa = \frac{\lambda_L}{\xi_{GL}} = \sqrt{\frac{m^2\beta}{2\mu_0\hbar^2 e^2}}$$

$\kappa < 1/\sqrt{2}$ Type I superconductor

$\kappa > 1/\sqrt{2}$ Type II superconductor

Superconductor-normal metal interface

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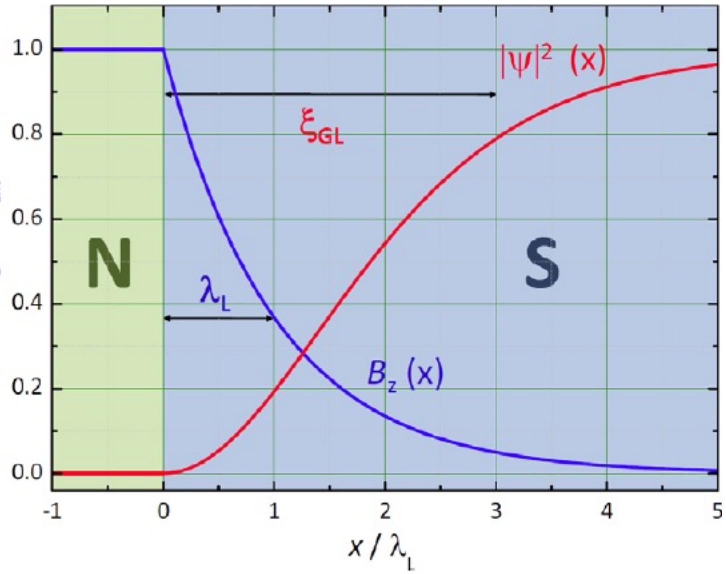
$$-\xi_{GL}^2\left(\frac{\boldsymbol{\nabla}}{i}+\frac{2e}{\hbar}\boldsymbol{A}\right)^2\psi+\psi-|\psi|^2\psi=0$$

to be solved (without magnetic field)

$$\xi_{GL}^2\frac{d^2\psi}{dx^2}+\psi-\psi^3=0$$

Solution:

$$\psi(x)=tanh\left(\frac{x}{\sqrt{2}\xi_{GL}}\right)$$



NS boundary energy

- **saving in field expulsion energy**

$$\Delta E_B \cong \frac{B_{ext}^2(T)}{2\mu_0} d_{eff}$$

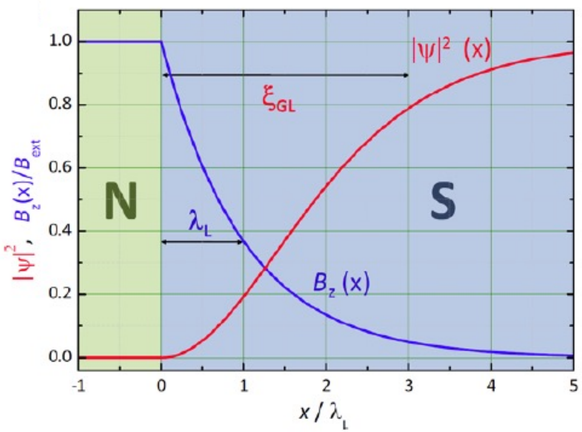
- **loss in condensation energy**

$$\Delta E_C \cong [g_n(T) - g_s(T)] \frac{V}{F} = \frac{B_{cth}^2(T)}{2\mu_0} \xi_{GL}$$

- **resulting boundary energy**

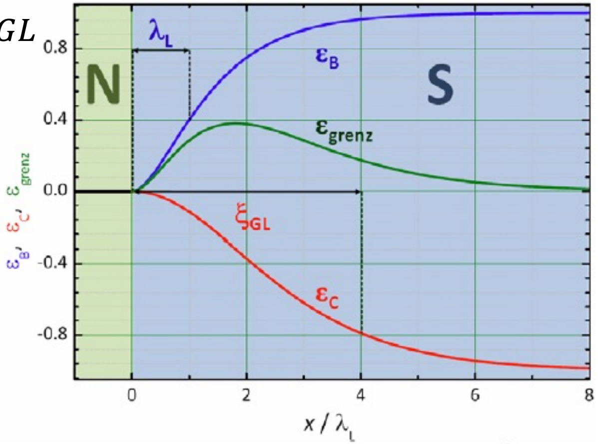
$$\Delta E_{grenz} = \Delta E_C - \Delta E_B$$

$$\cong \frac{B_{cth}^2(T)}{2\mu_0} \left\{ \xi_{GL} - \left(\frac{B_{ext}}{B_{cth}(T)} \right)^2 \lambda_L \right\}$$



for $\kappa = \frac{\lambda_L}{\xi_{GL}} < 1/\sqrt{2}$

boundary energy always positive

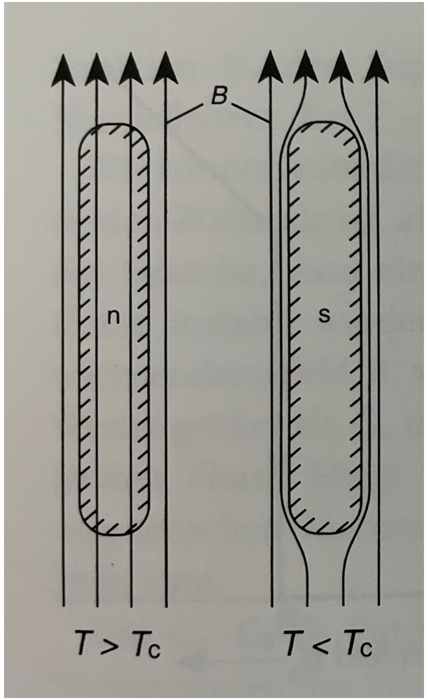


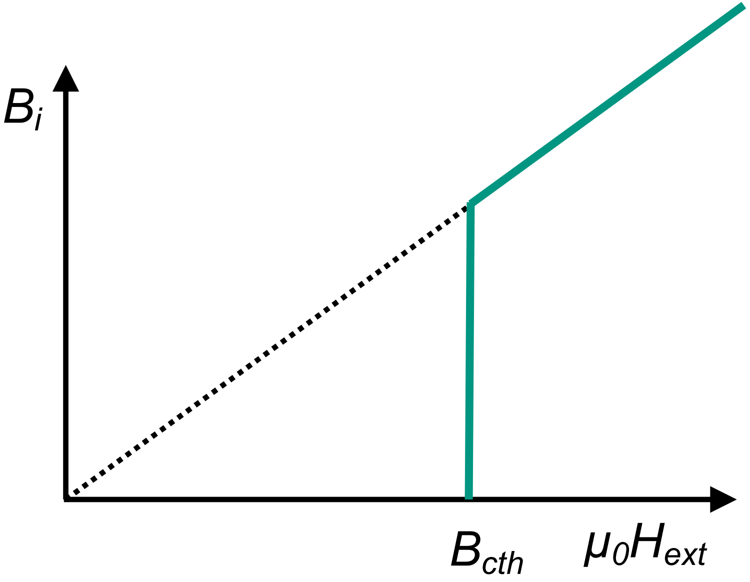
for $\kappa = \frac{\lambda_L}{\xi_{GL}} > 1/\sqrt{2}$

boundary energy gets negative at certain magnetic field

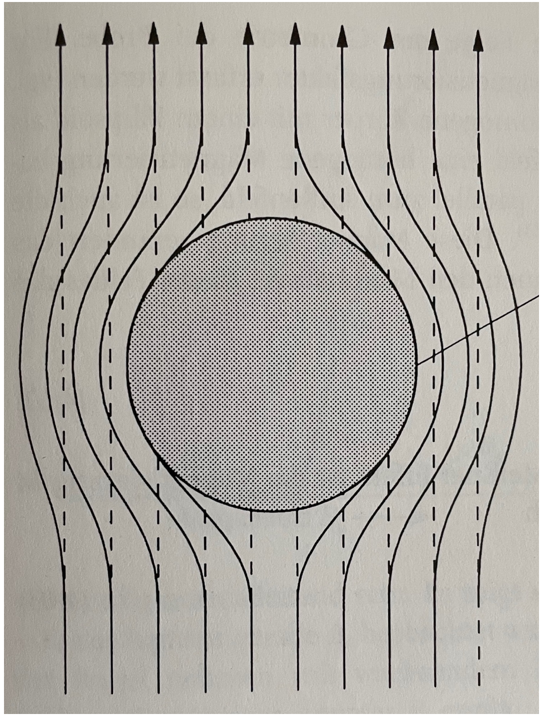
Intermediate state of type I superconductors

Meissner-Ochsenfeld effect

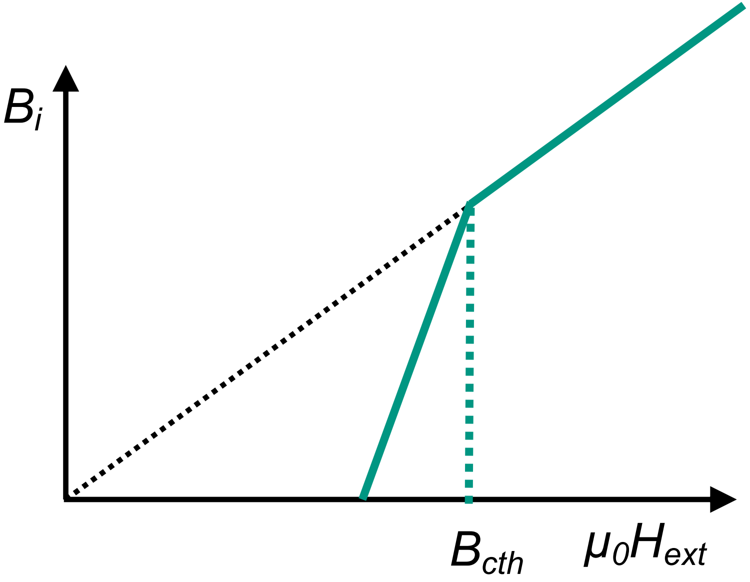




Intermediate state of type I superconductors



Effect of sample geometry



Intermediate state of type I superconductors

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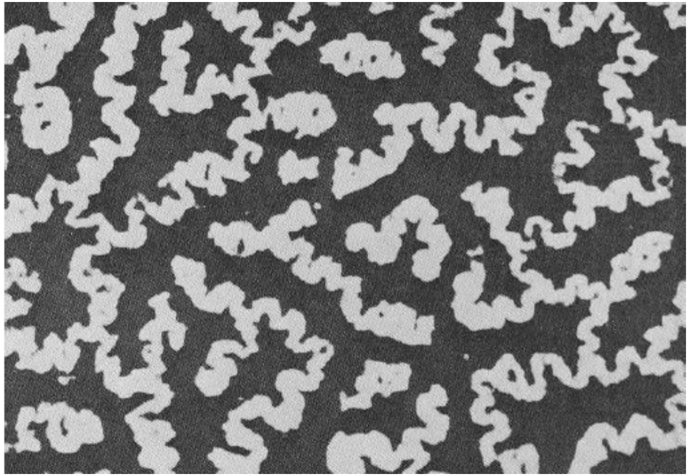
→ Intermediate state can result in quite complex structures

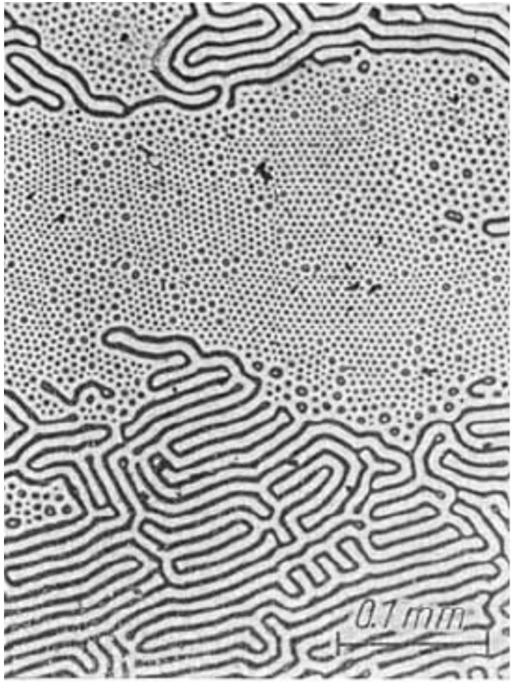
magnetooptical analysis of In and Ta plate
(sc: dark, nc: bright)



In

Henssler, Rinderer Helv. Phys. Acta **40**, 659 (1967)

Ta



U.Essmann et al. phys.stat.sol. (a) **43**, 151 (1977)